On regular hyperbolic fibrations

Deirdre Luyckx

Department of Pure Mathematics and Computer Algebra
Ghent University
Krijgslaan 281, S25
9000 Gent
Belgium
dluyckx@cage.ugent.be

Joint work with Matthew Brown, Gary Ebert

A hyperbolic fibration of $\mathsf{PG}(3,q)$ is a collection of $q-1$ hyperbolic quadrics $Q_i^+(3,q)$, $i=1,2,\dots,q-1$, and two lines L_0, L_∞ in $\mathsf{PG}(3,q)$ that partition the points of $\mathsf{PG}(3,q)$, see [1]. A hyperbolic fibration is called *regular* if the lines L_0 and L_∞ form a conjugate (skew) pair with respect to each of the polarities associated with the $q-1$ hyperbolic quadrics $Q_i^+(3,q)$ of the fibration. A regular hyperbolic fibration *agrees on* L_0, respectively L_∞, if the extension to $\mathsf{GF}(q^2)$ of each quadric $Q_i^+(3,q)$, $i=1,2,\dots,q-1$, meets L_0, respectively L_∞, in the same pair of points $\{x,\overline{x}\}$ which are conjugate with respect to the extension $\mathsf{GF}(q^2)$ of $\mathsf{GF}(q)$.

In [2], Baker, Ebert and Penttila show (algebraically) that a regular hyperbolic fibration that agrees on one of its lines corresponds to a flock of a quadratic cone in $\mathsf{PG}(3,q)$ with one conic (plane) specified, and conversely. For this correspondence, a unified geometric construction will be given for all q. Further it will be explained that all hyperbolic fibrations that agree on one of their lines, are necessarily regular. If q is even, an even stronger result holds: in this case it will be pointed out that all hyperbolic fibrations are regular.

References

[1] R. D. Baker, J. M. Dover, G. L. Ebert, and K. L. Wantz. Hyperbolic fibrations of $\mathsf{PG}(3,q)$. *European J. Combin.*, 20(1):1–16, 1999.

[2] R. D. Baker, G. L. Ebert, and Tim Penttila. Hyperbolic fibrations and q-clans. *Des. Codes Cryptogr.*, 34(2-3):295–305, 2005.